PRINCIPLES OF ECONOMETRICS

5TH EDITION

ANSWERS TO ODD-NUMBERED EXERCISES IN THE PROBABILITY PRIMER

EXERCISE P.1

(a) $\sum_{i=1}^{2} x_i = 18$

(b) $\sum_{t=1}^{3} x_t y_t = 87$

(c) $\bar{x} = 6$

(d) $\sum_{i=1}^{3} \left(x_i - \bar{x}\right) = 0$.

(e) $\sum_{i=1}^{3} \left(x_i - \bar{x}\right)^2 = 182$

(f) $\left(\sum_{i=1}^{3} x_i^2\right) - 3\bar{x}^2 = 182$

(g) $\sum_{i=1}^{3} \left(x_i - \bar{x}\right)\left(y_i - \bar{y}\right) = -3$

(h) $\sum_{j=1}^{3} x_j y_j - 3\overline{x}\overline{y} = -3$.

EXERCISE P.3

(a) $\sum_{i=1}^{3} \left(a - bx_i\right) = 3a - b\sum_{i=1}^{3} x_i$

(b) $\sum_{t=1}^{4} t^2 = 30$

(c) $\sum_{x=0}^{2} \left(2x^2 + 3x + 1\right) = 22$

(d) $\sum_{x=2}^{4} f\left(x+3\right) = f\left(5\right) + f\left(6\right) + f\left(7\right)$

(e) $\sum_{x=1}^{3} f\left(x, y\right) = f\left(1, y\right) + f\left(2, y\right) + f\left(3, y\right)$

(f) $\sum_{x=3}^{4} \sum_{y=1}^{2} \left(x + 2y\right) = 26$

EXERCISE P.5

(a) $P\left(SALES > 60000\right) = 0.0475$

(b) $P\left(40000 < SALES < 55000\right) = 0.7492$

(c) $SALES_{0.97} = 61,280$

(d) $P\left(PROFITS \le 0\right) = 0.0475$

EXERCISE P.7

(a) $E(SALES) = 25200$ cans

(b) $\text{var}(SALES) = 1{,}000{,}000 \text{ cans}^2$

(c) $P(SALES > 24000) = 0.8849$

(d) $PRICE_{0.95} = 231.55$ cents .

EXERCISE P.9

(a) The marginal distributions are

Political Party	Probability
Republican	0.45
Independent	0.15
Democrat	0.40

War Attitude	Probability
against	0.45
neutral	0.25
in favor	0.30

(b) $P(INDEPENDENT \mid IN\ FAVOR) = 0.167$

(c) They are not independent. For example

$$P(DEMOCRAT \text{ and } IN\ FAVOR) = 0 \neq P(DEMOCRAT) \times P(IN\ FAVOR) = 0.12$$

(d) $E(WAR) = 1.85$; $\text{var}(WAR) = 0.7275$

(e) $E(CONTRIBUTIONS) = 13.7$

standard deviation $(CONTRIBUTIONS) = 1.71$

EXERCISE P.11

(a) $P(VOTE = -1 \text{ and } PARTY = -1) = 0.3104$

(b) No, they are not statistically independent. For example,

$$P(VOTE = -1 \text{ and } PARTY = -1) = 0.3104 \neq P(VOTE = -1\ )P(PARTY = -1) = 0.14912$$

EXERCISE P.13

(a)

	W=0	*W*=1	*W*=2	*f*(*c*)
C=1	**0.06**	0.12	0.12	**0.3**
C=0	0.07	0.14	**0.49**	**0.7**
f(*w*)	**0.13**	**0.26**	0.61	

(b)

w	0	1	2
$f(w\|C=0)$	0.1	0.2	0.7

The conditional distribution $f(w|C=0)$ is not the same as $f(w)$, therefore the two random variables W and C are not statistically independent.

(c) $E(W)=1.48$ $\quad E(W|C=0)=1.6$ $\quad E(W|C=1)=1.2$

The LSU Tigers have a higher expected number of wins when the weather is not cold.

(d) $E(FOOD)=\$9{,}100$ $\quad$ standard deviation$(FOOD)=\$1374.77$

EXERCISE P.15

(a)

		Y		
		0	1	$f(x)$
	−20	**0.20**	0	**0.20**
X	0	**0.10**	0.15	0.25
	20	0.10	**0.45**	**0.55**
	$f(y)$	**0.40**	0.60	

(b) $E(X)=7$

The expected winnings are positive, so based on this criterion you should take the bet.

(c)

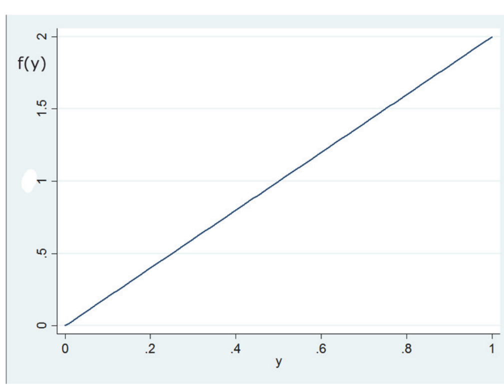
X	−20	0	20
$f(x\|Y=1)$	0	1/4	3/4

(d) $E(X|Y=1)=15$

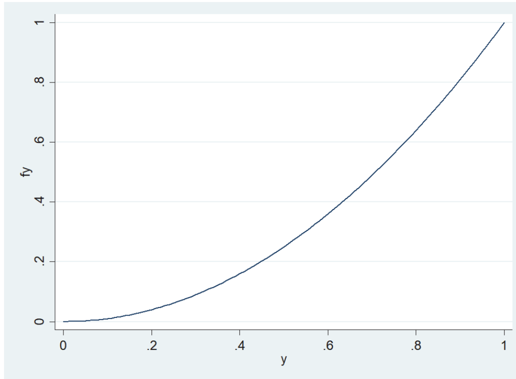
(e) $E(X)=E_Y\left[E(X|Y)\right]=E(X|Y=0)f_Y(0)+E(X|Y=1)f_Y(1)=-5(0.4)+15(0.6)=7$

EXERCISE P.17

(a)



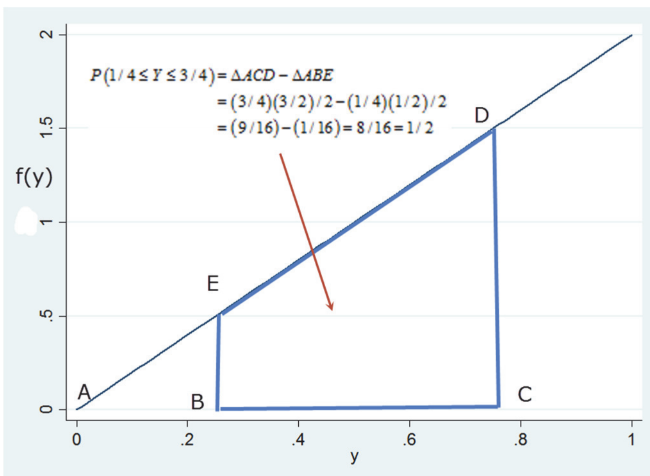
(b) $F(y) = y^2$



(c) $P(Y \le 1/2) = bh/2 = \left[(1/2)\times 1\right]/2 = 1/4$

(d) $P(Y \le 1/2) = F(1/2) = (1/2)^2 = 1/4$

(e)



(f) $P(1/4 \le Y \le 3/4) = F(3/4) - F(1/4) = (3/4)^2 - (1/4)^2 = 1/2$

EXERCISE P.19

(a) $E(Z) = (3/2)\mu$

(b) $\text{var}(Z) = (5/4)\sigma^2$

(c) $\text{var}(Z) = (3/4)\sigma^2$

(d) $\text{corr}(aX, bY) = \rho_{XY} = -0.5$

EXERCISE P.21

(a) $E(Y) = 3.5$ $E(Y^2) = 15.1667$ $\text{var}(Y) = 2.91667$

(b)

x	0	2	4	6
$f(x)$	1/2	1/6	1/6	1/6

$E(X) = 2$ $E(X^2) = 9.333$ $\text{var}(X) = 5.333$

(c)

y	1	2	3	4	5	6
$f(y\|x=0)$	1/3	0	1/3	0	1/3	0
$f(y\|x=2)$	0	1	0	0	0	0
$f(y\|x=4)$	0	0	0	1	0	0
$f(y\|x=6)$	0	0	0	0	0	1

(d) $E(Y \mid X = 0) = 3$ $E(Y \mid X = 2) = 2$ $E(Y \mid X = 4) = 4$ $E(Y \mid X = 6) = 6$

(e)

$z = xy$	0	4	16	36
$f(z)$	1/2	1/6	1/6	1/6

$$E(Z) = 28/3 = E(X^2)$$

(f) $\text{cov}(X,Y) = 2.333$

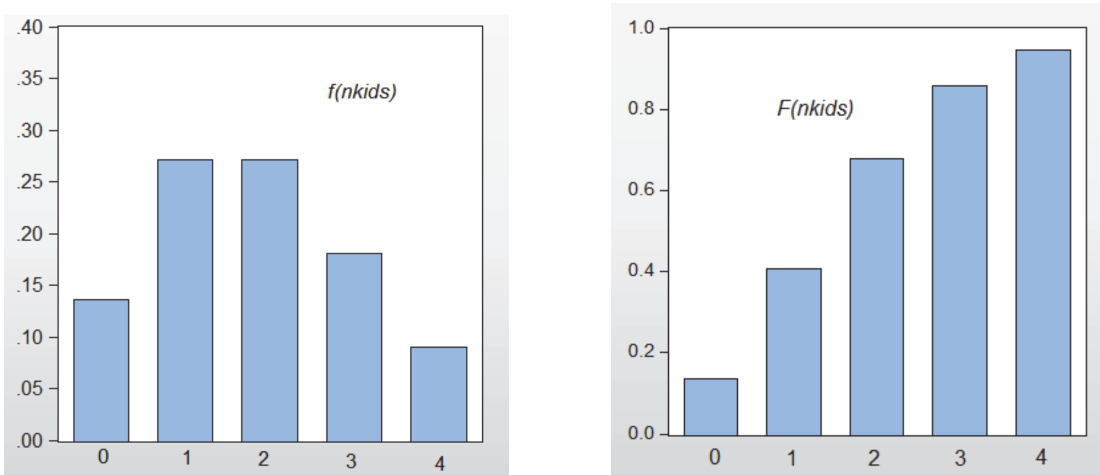
EXERCISE P.23

(a) *NKIDS* is a discrete random variable that takes on a "countable" number of values.

(b) and (c)

The *pdf* and *cdf* for *NKIDS* up to $NKIDS = 4$ are given in the following table and figures.

NKIDS	*pdf*	*cdf*
0	0.1353	0.1353
1	0.2707	0.4060
2	0.2707	0.6767
3	0.1804	0.8571
4	0.0902	0.9473



pdf f(nkids) and *cdf F(nkids)* for *NKIDS*

(d) $P(NKIDS > 1) = 0.594$

(e) $P(NKIDS \le 2) = 0.6767$

EXERCISE P.25

(a) $P(T = 1) = 0.149$ $\quad P(T = 4) = 0.168$

(b) $P(Y = 1 | T = 4) = 0.0036$

(c) $P[Y = 3, T = 4] = 0.049$

(d) $P(T = 4 | Y = 3) = 0.222$