

# Principles of Econometrics, 4<sup>th</sup> Edition

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  - B.1.2 Variance of a Discrete Random Variable
  - B.1.3 Joint, Marginal and Conditional Distributions
  - B.1.4 Expectations Involving Several Random Variables
  - B.1.5 Covariance and Correlation
  - B.1.6 Conditional Expectations
  - B.1.7 Iterated Expectations
- B.2 Working with Continuous Random Variables
  - B.2.1 Probability Calculations
  - B.2.2 Properties of Continuous Random Variables
  - B.2.3 Joint, Marginal and Conditional Probability Distributions
  - B.2.4 Iterated Expectations
  - B.2.5 Distributions of Functions of Random Variables
- B.3 Some Important Probability Distributions
  - B.3.1 The Bernoulli Distribution
  - B.3.2 The Binomial Distribution
  - B.3.3 The Poisson Distribution
  - B.3.4 The Uniform Distribution
  - B.3.5 The Normal Distribution
  - B.3.6 The Chi-square Distribution
  - B.3.7 The  $t$ -distribution
  - B.3.8 The  $F$ -distribution
- B.4 Random numbers
  - B.4.1 Uniform Random Numbers

## B.5 Exercises

# Appendix C Review of Statistical Inference

Learning Objectives

Keywords

C.1 A Sample of Data

C.2 An Econometric Model

C.3 Estimating the Mean of a Population

C.3.1 The Expected Value of  $\bar{Y}$

C.3.2 The Variance of  $\bar{Y}$

C.3.3 The Sampling Distribution of  $\bar{Y}$

C.3.4 The Central Limit Theorem

C.3.5 Best Linear Unbiased Estimation

C.4 Estimating the Population Variance and Other Moments

C.4.1 Estimating the Population Variance

C.4.2 Estimating Higher Moments

C.4.3 The Hip Data

C.4.4 Using the Estimates

C.5 Interval Estimation

C.5.1 Interval Estimation:  $\sigma^2$  Known

C.5.2 A Simulation

C.5.3 Interval Estimation:  $\sigma^2$  Unknown

C.5.4 A Simulation (continued)

C.5.5 Interval Estimation Using the Hip Data

C.6 Hypothesis Tests About a Population Mean

C.6.1 Components of Hypothesis Tests

C.6.2 One-tail Tests with Alternative "Greater Than" ( $>$ )

C.6.3 One-tail Tests with Alternative "Less Than" ( $<$ )

C.6.4 Two-tail Tests with Alternative "Not Equal To" ( $\neq$ )

C.6.5 Example of a One-tail Test Using the Hip Data

C.6.6 Example of a Two-tail Test Using Hip Data

C.6.7 The  $p$ -value

C.6.8 A Comment on Stating Null and Alternative Hypotheses

C.6.9 Type I and Type II Errors

C.6.10 A Relationship Between Hypothesis Testing and Interval Estimation

C.7 Some Other Useful Tests

C.7.1 Testing the Population Variance

C.7.2 Testing the Equality of Two Population Means

C.7.3 Testing the Ratio of Two Population Variances

C.7.4 Testing the Normality of a Population

C.8 Introduction to Maximum Likelihood Estimation (Contains some advanced material)

C.8.1 Inference with Maximum Likelihood Estimators

C.8.2 The Variance of the Maximum Likelihood Estimator

C.8.3 The Distribution of the Sample Proportion

C.7.4 Asymptotic Test Procedures

C.8.4a The Likelihood Ratio (LR) Test

C.8.4b The Wald Test

C.8.4c The Lagrange Multiplier (LM) Test

C.9 Algebraic Supplements (Optional)

C.9.1 Derivation of Least Squares Estimate

C.9.2 Best Linear Unbiased Estimation  
C.10 Kernel Density Estimator  
C.11 Exercises

## **Appendix D Tables**

Table 1 Cumulative Probabilities for the Standard Normal Distribution

Table 2 Percentiles for the  $t$ -Distribution

Table 3 Percentiles for the Chi-Square Distribution

Table 4 95<sup>th</sup> Percentile for the  $F$ -Distribution

Table 5 99<sup>th</sup> Percentile for the  $F$ -Distribution